Countervailing Incentives in Optimal Procurement Auctions

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February 27, 1997

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Abstract

Highly specialized and costly goods and services, for which there are no active markets and only one buyer, are often purchased by means of procurement auctions, which result in the award of a (possibly) cost-based incentive contract to produce the good. These contracts are generally characterized by moral hazard, which induces firms to overstate their costs or refrain from efficient cost-reducing effort, and by adverse selection, whereby more efficient firms have better outside opportunities and thus less incentive to participate. This paper shows that in a simple procurement model, the optimal policy for the buyer is to choose the bidder who minimizes a given “adjusted cost” function. The chosen firm is paid a price which includes a positive information rent, which decreases as the firm’s cost increases. This solution balances the countervailing incentives for the bidders to overstate low costs in order to share in an incentive fee, and to understate high costs in order to increase one’s probability of winning the contract.
1 Introduction

Highly specialized and costly goods and services, for which there are no active mar-
kets, often only one buyer, and uncertain costs related to development of a unique
product, are often purchased by means of a procurement auction. In this procedure,
a buyer specifies characteristics of the product desired, and prospective sellers sub-
mitt bids specifying a price for the good or an estimate of the cost of producing it,
and in some cases more specific characteristics of the product they intend to deliver.
One seller is then selected, and awarded a contract (which may contain cost-based
incentives) to produce and deliver the good.

Procurement auctions are often used by governments for purchasing goods of
varying degrees of specialization, from items as commonplace as roads and bridges
to those as unique as satellites, fighter planes, and even research programs. Auctions
are also an appropriate model for some of the procedures used by private firms for
purchasing specialized tools and components, as well as specialized services, such as
consulting, advertising, and design work. One of the most widely known and widely
studied uses of procurement auctions is that of the U.S. Department of Defense for
the acquisition of advanced communications, observation, and weapons systems. Such
acquisitions accounted for over $180 billion per year in the mid 1980’s, and account
for over $140 billion per year today, or about 60% of the U.S. Defense budget. In
recent years, a portion of this amount was even spent to procure research by academic
economists into how to implement procurement auctions more efficiently. Bower and
Dertouzos (1994) present a collection of some of this research.

The focus of this paper will be on procurement of unique goods for which
development is necessary. These purchases are characterized by three features of
interest: (1) moral hazard, which can affect ex post cost measurements (the successful
bidder may “run up” the costs if payment is cost-contingent, and choose whether to
engage in efficient cost-reducing effort); (2) uncertainty as to the cost distributions
of prospective sellers (the buyer does not know which firm is most efficient); and (3) adverse selection, which can result in choosing a high-cost firm (since prospective sellers have private information about their costs and outside opportunities).

The key problem for the buyer is to determine what type of auction design, and what type of contract incentives (if any), will generate the best product for the lowest expected expenditure. The auction design is, at its simplest, a three-step process: First, the buyer announces the type of product desired and the form of the contract being offered. Second, prospective bidders who choose to bid submit sealed prices or cost estimates. Third, the buyer awards the contract, usually to the lowest bidder, but sometimes to another bidder based on other information submitted with the bid.

Of course, the actual procedure is often more complicated, with the complexity of the process reflecting that of the good to be procured, and the amount of research and development necessary to produce it. Generally, the bids include not only a price or cost estimate, but also a plan for producing the good or a preliminary design. Often, contracts for basic research, design and development, prototypes, and production models (for goods procured in multiple units) are awarded separately, each with a different incentive structure, even if the different phases of the project are undertaken by the same firm.

The plan of the paper is as follows. Section 2 describes the forms of standard contracts, and reviews some of the relevant literature analyzing these contracts in the auction framework. Section 3 describes the auction model and characterizes the set of feasible contracts. Section 4 derives the optimal auction in this model, and gives examples to illustrate how it performs in the framework of McCall (1970) and McAfee and McMillan (1986). Section 5 concludes and suggests further work.


2 Background

2.1 Institutional Characteristics

There are three types of procurement contracts in common use, although many others could be imagined. Each type of contract has its own drawbacks, and is suitable (or unsuitable) for different circumstances. The simplest is the firm fixed-price (FFP) contract, in which bidders submit proposed prices for the good, and the winner of the contract is paid the amount of the bid. This presents no problem of moral hazard, since the seller’s realized costs are never measured and thus have no *ex post* impact on the price. However, the seller is required to bear all the risk associated with cost uncertainty, so this may be unsuitable in the presence of high cost uncertainty if the prospective sellers are significantly more risk-averse than the buyer.

At the other extreme is the cost-plus-fixed-fee (CPFF) contract, in which bidders submit estimates of the (physical) cost they will incur to produce the product. The winner is (usually) the one who submits lowest cost estimate, and is then paid a price based on audited, realized costs, plus a fixed fee which is usually based on a percentage of the initial cost estimate. This contract avoids the problem of risk-averse bidders, since it places the entire cost risk on the buyer. However, it is fraught with moral hazard, since, *ex post*, the seller has no incentive to reduce costs (since the effort required to do so may be costly), and the seller or its managers may even be able to capture private benefits in ways that increase the realized costs but do not reduce the “plus” fee. For example, they may be able to allocate costs of other activities, which generate revenue in other markets, to the contract cost. In extreme cases, they may increase salaries of managers and employees, purchase unnecessarily luxurious office equipment, or engage in other forms of “cost padding.” Most important, however, the limitation of profit to a fixed percentage of the initial cost estimate may discourage the more profitable, lower cost firms from bidding at all.
In between these two extremes is the cost-plus-incentive-fee (CPIF) contract, and the similar fixed-price-incentive (FPI) contract, in which bidders submit cost estimates, and the winner is paid an amount determined by both the bid and the audited, realized cost, so as to capture a portion of the benefits of cost reduction, or suffer a like portion of a cost overrun. Typically, the price is $C + \alpha P + \beta (P - C)$, where $C$ is the audited, realized cost, $P$ is the bid, or “target” cost, $\alpha > 0$ is a percentage of the bid representing “target” profit, and $\beta \in (0, 1)$ is the “risk-sharing parameter,” or the amount of the cost deviation gained or lost by the seller ($\beta = 1$ corresponds to the FFP contract, and $\beta = 0$ to the CPFF contract). Typically $\alpha$ and $\beta$ are set by the buyer in advance of bidding, although in some cases $\beta$ is subject to negotiation or bidding.

Often, different types of contracts may be used for different phases of the development and production of a complex good. For example, Smith, Shulman, and Leonard (1996) give a fairly detailed account of the acquisition process used by the U.S. Air Force for the F-117 “stealth” fighter aircraft. Over the course of the ten-year development and production process, all three types of contracts were used: CPFF for the initial development phase, which involved basic research and had the most cost uncertainty; FPI for the subsequent development phase, when the problem was more well-defined and there was less cost risk; FPI again for the first 28 production units, when production costs were still fairly uncertain; and FFP for the next 31 production units, when the cost uncertainty was much lower.

### 2.2 Existing Literature

Many of the difficulties with the standard contracts are widely known and documented in the literature. The most obvious problem with the CPFF contract is that if the contract is awarded to the lowest bidder, the bidders’ incentives ex ante are to submit unrealistically low bids, and the winner’s ex post incentive is at best,
to engage in no cost-reducing effort, and at worst, to capture private benefits in a manner that increases the (apparent) cost as much as possible. Furthermore, Lucas (1994) shows that adverse selection causes the firms which are least efficient (in terms of outside opportunities, i.e., generating market profits) to be the most likely to win the contract.

The FFP contract, on the other hand, has no such problems, but for products involving a large degree of uncertainty (for example, projects which have large research and development components) it is possible that all firms will be too risk-averse to be willing to bid. This is frequently the case for basic research or design projects, such as the design of an aircraft or satellite to do a specified job, since there may be very little prior knowledge of what type of product will ultimately be required to do the job. It is also the case when the amount of work required may vary greatly based on factors not entirely under the control of the contractor, as is the case for some consulting services and for complex legal work.

The CPIF contract would appear to balance the risk-sharing and moral hazard problems so that an optimal tradeoff could be achieved by appropriate choice of the sharing parameter $\beta$. However, McCall (1970) shows that if the winner is chosen to be the one submitting the lowest bid (target cost), then under fairly general conditions, the firm with the highest cost distribution is most likely to win the contract. The reason is that high-cost firms with lower outside profit opportunities bid below their expected cost and “plan” to overrun and accept a lower rate of profit, and low-cost firms with higher outside profit opportunities bid above their expected cost and plan to make higher profits from the incentive fee. Amazingly, the buyer’s optimal policy under the McCall model is to award the contract to the highest bidder, providing the bid is high enough. Unfortunately, this works only if, when firms submit their bids, they believe the contract will be awarded to the lowest bidder. Needless to say, this situation is not possible in the real world where buyers will buy again in the future, since the buyer cannot repeatedly both credibly commit to choosing the lowest bidder,
but actually choose the highest bidder.

Canes (1975) points out that this problem can be partially, though not completely, mitigated by allowing firms to submit a two-dimensional bid, consisting of both a target cost $P$ and a sharing rate $\beta$. In this case, relatively inefficient firms, those whose outside opportunity revenue $R$ exceeds their cost of production $c_i$, and those efficient firms with $R - c_i < \alpha c_i$ will submit bids, but the most efficient firms, those with $R - c_i > \alpha c_i$, will not submit any bid since their (IR) constraint will not be satisfied.

Under the assumption that the lowest cost estimate wins, McCall finds that the winning firm receives as a transfer exactly its outside opportunity revenue if this is greater than its expected cost, and exactly its expected costs otherwise. Thus, any firm with a cost in a large interval will receive the same transfer, regardless of its actual cost level. This pooling is similar in character to that in the countervailing incentives model of Lewis and Sappington (1989a, 1989b). In their model, the individual rationality (IR) constraint (nonnegative profits) binds for the highest and lowest possible costs, and there is pooling (identical transfer for different costs) for a (possibly) nontrivial interval of costs which may be strictly between the highest and lowest possible costs. In both models, pooling arises from countervailing incentives: the incentive to overstate low cost realizations (so as to “underrun” and collect a higher incentive fee) and understate high cost realizations (thus planning an “overrun,” but having a greater probability of winning the contract). It will be shown later that this pooling may occur in all feasible implementations of the auction, and it does in fact occur in the optimal auction.

Using a similar model, McAfee and McMillan (1986) assume that costs are subject to randomness and to cost-reducing effort (which itself is costly), and find that the CPIF contract “usually” minimizes procurement expenditure and induces bidders to reveal their costs. This is seemingly the opposite of McCall’s (1970) result
(albeit under different assumptions), and McAfee and McMillan appear to be unaware of McCall’s paper. In McAfee and McMillan’s model, it is implicitly assumed that opportunity costs are observable and contractible, just like physical costs. This is actually equivalent to setting outside revenue equal to zero in McCall’s model, and once the adjustment is made the two models make identical predictions.

Holt (1979) finds that under reasonable assumptions, expected procurement cost is independent of the target profit rate $\alpha$, but increasing in the sharing rate $\beta$. In particular, he shows that for the CPIF contract, if the bidders are risk-averse, the production cost is unknown but the distribution is the same for all potential bidders, there are no opportunities for reducing cost through effort, and outside opportunities are privately known but $ex$ $ante$ independent and identically distributed, then the bidders’ expected Von Neumann-Morgenstern utility of profit is independent of the parameters $\alpha$ and $\beta$. An increase in the risk-sharing parameter $\beta$ causes an increase in bids, an increase in expected profits (though not expected utility of profits, as above), and an increase in expected procurement expenditure. This is consistent with the notion of the buyer bearing more risk as $\beta$ increases. On the other hand, an increase in the target profit rate $\alpha$ causes lower bids (with the same cost distribution) and thus more “overruns” (i.e., $ex$ $post$ production cost in excess of bid), but no change in expected procurement expenditure (price). One consequence of this is that the presence of “overruns” does not necessarily indicate that the buyer is overpaying; instead, it may be that the bids are systematically less than the true cost.

In a different paper, Holt (1980), using the same assumptions, derives the optimal bid under the assumption of symmetric Nash equilibrium and shows that if bidders are risk-averse (neutral), the expected expenditure is lower (equal) with a low-(first-)price sealed-bid auction than with a second-price auction.

Harris and Raviv (1981) prove essentially the same result in the case of selling auctions (as opposed to procurement auctions), as a special case of a more general
framework in which the set of possible reservation prices may be discrete or continuous, and the number of units available for sale may be greater than one (as, for example, in the case of U.S. Treasury bill auctions).

There are many other articles which solve specialized problems related to auctions in the above framework. Bower and Osband (1991) discuss a model with very restrictive assumptions. Risk-neutral firms with certain costs (and possibilities for cost-reducing effort) bid for a FFP research and development contract, which will be followed by a CPFF production contract for the same firm. Cost is audited after development but before production, and the profit rate based on these costs is fixed and common knowledge at the beginning. They conclude that the excess profits generated in the second period will be dissipated by the competitive bidding process in the first period, and discuss the effect of the second-period profit rate on total expenditures.

Smith, Shulman, and Leonard (1996) describe in detail the acquisition of the F-117 fighter, in which the development contract was CPFF, and production contracts were CPIF and FFP. (Note that this is the opposite of the case treated by Bower and Osband.) In this program, the buyer (the U.S. Air Force) used a different management technique that in previous Air Force acquisition programs. For example, the they used a smaller management team, and implemented a more flexible policy with respect to design changes, than had been used in the past. The authors find that the new management strategy resulted in significant cost saving compared with other similar programs, in the sense that the F-117 was a higher quality product which required substantial development of new technology, yet was acquired for a price comparable to that for other similar systems.

Che (1993, 1994) develops a model in which firms submit multi-dimensional bids, specifying both price and a vector of characteristics. The buyer evaluates each bid based on a hedonic utility function (“scoring rule”) as well as the offered price, possibly engaging in negotiation with bidders to refine their bids. This model is
similar to the actual procedure used in U.S. Department of Defense acquisitions. (The DoD procedure is described in detail from the bidder’s point of view in Sammet and Green (1990).) Che considers three types of auctions based on this model: first-score, second-score, and second-preferred offer. All three auctions yield the same expected utility, and the first two yield the first-best level of quality, but this is higher than the optimal level indicated by the revelation principle.

Manelli and Vincent (1995) suggest that in many procurement environments, auction mechanisms are necessarily suboptimal, because adverse selection may result in only low-quality items being offered in the auction. They seek to develop criteria for deciding when it is best for the buyer to run an auction, and when it is best to negotiate sequentially with prospective sellers.

3 The Auction Model

This section presents a model of the procurement process as an auction in the standard principal-agent framework, with the buyer as the principal and the prospective bidders as agents. We will compare the results of our model with those of McCall (1970).

McCall’s model assumes that firms know their own outside opportunities (which are drawn from independent identical distributions), and that costs of producing the good are random and distributed identically for all firms. McCall also assumes that firms submit bids (cost estimates) calculated to equate profits from the contract with profits available from exercising the outside opportunity. This implicitly assumes that there is a firm at every possible point in the support of the cost distribution, or equivalently, that there is an infinite number of bidders. Under the assumption that the contract is always awarded to the firm which submits the lowest cost estimate, McCall finds that the winning firm receives as a transfer exactly its
outside revenue opportunity if this is greater than its expected cost, and exactly its expected costs otherwise. Thus, if the winning firm’s outside revenue would be $R$, and its cost of production is $c$, it earned a profit of $R - c$ if $c < R$ and zero otherwise.

### 3.1 Notation and Assumptions

Let the following notation be defined:

\[ \begin{align*}
V & = \text{The principal’s valuation of the good to be procured} \\
N & = \text{Number of firms which are potential bidders (agents)} \\
c_i & = \text{Firm $i$’s physical cost of producing the good; } c = (c_1, \ldots, c_N) \\
\tilde{c}_i & = \text{Firm $i$’s reported cost estimate; } \tilde{c} = (\tilde{c}_1, \ldots, \tilde{c}_N); \tilde{c}_{\sim i} \text{ denotes the cost reports of all firms except firm $i$} \\
K(c_i) & = \text{Outside opportunity profit for a firm whose (true) physical cost is } c_i \\
T_i(\tilde{c}_i|\tilde{c}_{\sim i}) & = \text{Payment (transfer) received by a firm $i$, when cost reports are } \tilde{c} \\
Q_i(\tilde{c}_i|\tilde{c}_{\sim i}) & = \text{Probability of firm $i$ being asked to produced the good, when cost reports are } \tilde{c} \\
\pi_i(\tilde{c}_i|c_i, \tilde{c}_{\sim i}) & = \text{Firm $i$’s profit from reporting cost } \tilde{c}_i \text{ when the firm’s true cost is } c_i \text{ and the other firms’ reports are given by } \tilde{c}_{\sim i} \\
\pi_i(c_i) & \equiv \pi_i(c_i|c); \text{ That is, firm } i \text{’s profit from truthfully reporting cost } c_i, \text{ given that all other firms also report truthfully and have costs } c.
\end{align*} \]

Note that from the above definitions, firm $i$’s net profit is:

\[ \pi_i(\tilde{c}_i|c_i, \tilde{c}_{\sim i}) = Q_i(\tilde{c}_i|\tilde{c}_{\sim i})[T_i(\tilde{c}_i|\tilde{c}_{\sim i}) - (K(c_i) + c_i)] \]

We may assume, without loss of generality, that there are lower and upper limits to cost, i.e., $c_{\text{min}}$ and $c_{\text{max}}$, such that $c_i \in [c_{\text{min}}, c_{\text{max}}]$ $\forall i = 1, \ldots, N$. Cost cannot be negative, so $c_{\text{min}} \geq 0$. The assumption of an upper limit is without loss
of generality, since if the cost of the good is greater than the principal’s valuation $V$, then the good will not be procured. Thus, we are only concerned with cases where $c_{\text{max}} \leq V$. We assume that each firm’s cost $c_i$ is drawn from a distribution with a continuous density function $f(c_i) > 0$ on $[c_{\text{min}}, c_{\text{max}}]$. The corresponding cumulative distribution function will be denoted by $F(c_i)$. The distribution is assumed to satisfy the monotone hazard rate condition (MHRC), that is, $f(c_i) \left(1 - F(c_i)\right)$ is nondecreasing for all $c_i \in [c_{\text{min}}, c_{\text{max}}]$. (We will use this assumption in Section 4.)

The functions $T_i$, $Q_i$, and $K$ are assumed to be continuous, and differentiable almost everywhere. All the $T_i$ are identical except for the index of the firm whose transfer is given, and likewise for the $Q_i$. We will use $T'_i(c_i)$ and $Q'_i(c_i)$ to denote the derivatives $\frac{\partial T_i(c_i|\tilde{c}_{-i})}{\partial c_i}$ and $\frac{\partial Q_i(c_i|\tilde{c}_{-i})}{\partial c_i}$, respectively. We will use circumflexes (“hats”) to denote expected value of a function conditional on firm $i$’s own cost; that is,

$$
\hat{T}_i(\tilde{c}_i) \equiv E[T_i(\tilde{c}_i|\tilde{c}_{-i})|c_i]
$$

$$
\hat{Q}_i(\tilde{c}_i) \equiv E[Q_i(\tilde{c}_i|\tilde{c}_{-i})|c_i]
$$

$$
\hat{\pi}_i(\tilde{c}_i|c_i) \equiv E[\pi_i(\tilde{c}_i|c_i, \tilde{c}_{-i})|c_i]
$$

$$
\hat{\pi}_i(c_i) \equiv E[\pi_i(c_i)|c_i]
$$

Since $Q_i$ represents a probability, its value will be between 0 and 1. Since $T_i$ represents a transfer from the buyer to the seller, it will be between 0 and the buyer’s valuation $V$. Since $Q_i$ and $T_i$ are both continuous almost everywhere and bounded, $\hat{T}_i$, $\hat{Q}_i$, and $\hat{\pi}_i$ are continuous and differentiable almost everywhere.

Each firm’s outside opportunity is assumed to be nonnegative, i.e., $K(c_i) \geq 0$, and is a strictly decreasing function of its physical cost, i.e., $K'(c_i) < 0$ (except when $K(c_i) = 0$, in which case $K'(c_i) = 0$). These are consistent with the assumptions of both Lewis and Sappington (1989b), in which the $K(c_i)$ is considered to be a fixed cost negatively correlated with variable cost, and McCall (1970), in which $K(c_i) = \max(R - c_i, 0)$ where R is a fixed outside revenue (identical for all firms) and the 0 represents the case when the outside revenue is less than cost, so the firm will chose
to do nothing if it does not win the auction. Lewis and Sappington assume that $K(c_1)$ is strictly concave, which would exclude the McCall model. We will not need this strong assumption, and would like to include the McCall model, so we assume only that $K(c_1)$ is weakly concave, i.e., that $K''(c_1) \leq 0$. (In McCall, $K''(c_1) = 0$ everywhere it is defined.) In addition, we assume that $K'(c_1) \geq -1$, that is, each firm’s opportunity profit does not decrease more than one unit for every unit of cost increase.

We assume first that firms are risk-neutral. This implies that without loss of generality, we may consider $T_i(\tilde{c}_i|\tilde{c}_{-i})$ to be the expected value of the payment to firm $i$, based on its probability $Q_i(\tilde{c}_i|\tilde{c}_{-i})$ of being selected, regardless of whether or not it actually is selected. That is, Eq. (1) becomes

$$
\pi_i(\tilde{c}_i|c_i, \tilde{c}_{-i}) = T_i(\tilde{c}_i|\tilde{c}_{-i}) - Q_i(\tilde{c}_i|\tilde{c}_{-i})(K(c_i) + c_i)
$$

and the expected profit is

$$
\hat{\pi}_i(\tilde{c}_i|c_i) = \hat{T}_i(\tilde{c}_i) - \hat{Q}_i(\tilde{c}_i)(K(c_i) + c_i)
$$

This makes the notation for the risk-neutral case simpler; it will have to be abandoned in the case of risk-averse bidders.

The principal’s (i.e., the buyer’s) optimization problem may thus be written as

$$
\max_{Q_i(\cdot), T_i(\cdot)} E \left[ V \sum_{i=1}^{N} Q_i(\tilde{c}_i|\tilde{c}_{-i}) - \sum_{i=1}^{N} T_i(\tilde{c}_i|\tilde{c}_{-i}) \right]
$$

subject to:

$$
\hat{\pi}_i(c_i) \geq 0 \quad \text{(IR)}
$$

$$
\hat{\pi}_i(c_i) \geq \hat{\pi}_i(\tilde{c}_i|c_i) \quad \forall \tilde{c}_i \in [c_{\min}, c_{\max}] \quad \text{(IC)}
$$

$$
Q_i(c_i|\tilde{c}_{-i}) \geq 0 \quad \forall \, i = 1, \ldots, N
$$

$$
\sum_{i=1}^{N} Q_i(\tilde{c}_i) \leq 1
$$

where the objective function is the principal’s valuation times the probability she procures the good (i.e., her expected valuation) minus the transfers paid to the agents.
The (IR) constraints guarantee that all agents are willing to participate, and the (IC) constraints guarantee that we can restrict attention to mechanisms in which agents truthfully reveal their costs, as per Myerson’s (1981) Revelation Principle.

3.2 Characterization of Feasible Contracts

We now state and prove a series of propositions which characterize the feasible solutions to (BP) (i.e., the feasible contracts), and then solve for the optimal contract.

Proposition 1 shows that for all feasible contracts, a firm’s profit is a weakly decreasing function of its physical cost of production. This corresponds to what we expect is the case in the “real world,” that is, that firms with lower costs generally make higher profits. It also corresponds to Lewis and Sappington’s conclusion that “An agent’s expected rents under any incentive scheme that is optimally designed in the presence of adverse selection are generally greater the greater is the agent’s level of ability.”

**Proposition 1.** In any feasible solution to (BP),

\[ \hat{\pi}'(c_i) = -\hat{Q}_i(c_i)(K'(c_i) + 1) \leq 0 \]

almost everywhere.

*Proof.* From Eq. (3), we have

\[ \hat{\pi}_i(c_i) = \hat{\pi}_i(c_i|c) = \hat{T}_i(c_i) - \hat{Q}_i(c_i)(K(c_i) + c_i) \]

Differentiating, we have

\[ \hat{\pi}'_i(c_i) = \hat{T}'_i(c_i) - \hat{Q}'_i(c_i)(K(c_i) + c_i) - \hat{Q}_i(c_i)(K'(c_i) + 1) \]

(4)

Since \( \hat{T}_i, \hat{Q}_i, \) and \( K \) are continuous and differentiable almost everywhere, (IC) implies

\[ \frac{\partial \hat{\pi}_i(\tilde{c}_i|c_i)}{\partial \tilde{c}_i} \bigg|_{\tilde{c}_i=c_i} = 0 \]
for all $\tilde{c}_i \in (c_{\min}, c_{\max})$. That is,
\[
\hat{T}'_i(c_i) - \hat{Q}'_i(c_i)(K(c_i) + c_i) = 0 \quad \text{and} \quad \hat{T}'_i(c_i) - \hat{Q}'_i(c_i)(K(c_i) + c_i) = 0
\]  
(5)

We can now substitute 0 for $\hat{T}'_i(c_i) - \hat{Q}'_i(c_i)(K(c_i) + c_i)$ in Eq. (4) to obtain:
\[
\pi'_i(c_i) = -\hat{Q}_i(c_i)(K'(c_i) + 1)
\]  
(6)

This proves the equality part of the proposition. To see that this is nonpositive, recall from our assumptions that $-1 \leq K'(c_i) \leq 0$, so $K'(c_i) + 1 \geq 0$. $\hat{Q}_i(c_i)$ is constrained to be nonnegative, since it is the expected value of $Q_i(c_i|c_{-i})$, which represents a probability. The minus sign in front, then, means that the entire right-hand side of Eq. (6) must be nonpositive.

Proposition 2 shows that for all feasible auctions, a firm’s probability of being chosen to produce the good never increases if its cost of production increases. This also corresponds to a desirable “real world” property, that is, that firms with higher costs are (weakly) less likely to be chosen.

**Proposition 2.** In any feasible solution to (BP), $\hat{Q}_i(c_i)$ is nonincreasing in $c_i$.

**Proof.** (IC) \Rightarrow
\[
\hat{\pi}_i(c_i) - \hat{\pi}_i(\tilde{c}_i|c_i) = \hat{T}_i(\tilde{c}_i) - \hat{Q}_i(\tilde{c}_i)(K(c_i) + c_i)
\]
\[
= \hat{T}_i(\tilde{c}_i) + \left\{ -\hat{Q}_i(\tilde{c}_i)(K(\tilde{c}_i) + \tilde{c}_i) + \hat{Q}_i(\tilde{c}_i)(K(\tilde{c}_i) + \tilde{c}_i) \right\} - \hat{Q}_i(\tilde{c}_i)(K(c_i) + c_i)
\]
\[
= \hat{\pi}_i(c_i) + \hat{Q}_i(\tilde{c}_i)(K(\tilde{c}_i) + \tilde{c}_i) - \hat{Q}_i(\tilde{c}_i)(K(c_i) + \tilde{c}_i) - \hat{Q}_i(\tilde{c}_i)(K(c_i) + c_i)
\]

The above inequality implies
\[
\hat{\pi}_i(c_i) - \hat{\pi}_i(\tilde{c}_i) \geq -\hat{Q}_i(\tilde{c}_i)(K(c_i) + c_i) + \hat{Q}_i(\tilde{c}_i)(K(\tilde{c}_i) + \tilde{c}_i)
\]  
(7)
Reversing the roles of $c_i$ and $\tilde{c}_i$ and multiplying through by $-1$, we have

\[ \hat{\pi}_i(c_i) - \hat{\pi}_i(\tilde{c}_i) \leq -\hat{Q}_i(c_i)(K(c_i) + c_i) + \hat{Q}_i(c_i)(K(\tilde{c}_i) + \tilde{c}_i) \]  

(8)

Combining (7) and (8) and factoring out the $\hat{Q}_i$ terms,

\[ \hat{Q}_i(c_i)[K(\tilde{c}_i) - K(c_i) + \tilde{c}_i - c_i] \geq \hat{Q}_i(\tilde{c}_i)[K(\tilde{c}_i) - K(c_i) + \tilde{c}_i - c_i] \]

Since $-1 \leq K'(c_i) \leq 0$, we know that $|K(\tilde{c}_i) - K(c_i)| \leq |\tilde{c}_i - c_i|$, so the sign of $(\tilde{c}_i - c_i)$ is the sign of the entire parenthetical expression. In particular, if $(\tilde{c}_i - c_i)$ is positive, we can divide through by the parenthetical expression, leaving $\hat{Q}_i(c_i) \leq \hat{Q}_i(\tilde{c}_i)$. In other words,

\[ \tilde{c}_i > c_i \Rightarrow \hat{Q}_i(\tilde{c}_i) \leq \hat{Q}_i(c_i) \]

So $\hat{Q}_i(c_i)$ is nonincreasing in $c_i$.

Having shown that (IC) implies Propositions 1 and 2, we now show that the converse is also true.

**Proposition 3.** Any allocation which satisfies Propositions 1 and 2 also satisfies (IC).

**Proof.** Suppose this is not true. That is, suppose there exists a contract such that Propositions 1 and 2 are satisfied, and $\hat{\pi}_i(c_i|c_i) < \hat{\pi}_i(\tilde{c}_i|c_i)$ for some $c_i$ and $\tilde{c}_i$. Then

\[ \hat{\pi}_i(c_i|c_i) - \hat{\pi}_i(\tilde{c}_i|c_i) < 0 \]

which implies

\[ \int_{c_i}^{\tilde{c}_i} \hat{\pi}_i(s|c_i) ds < 0 \]  

(9)

where $\hat{\pi}_i(s|c_i)$ denotes the partial derivative $\frac{\partial}{\partial s} \hat{\pi}_i(s|c_i)$. 

15
Now,
\[
\hat{\pi}'(c_i) = \frac{\partial}{\partial s} \hat{\pi}_i(s|c_i) \bigg|_{s=c_i} + \frac{\partial}{\partial s} \hat{\pi}_i(c_i|s) \bigg|_{s=c} \\
= \hat{\pi}_i(c_i|c_i) + \frac{\partial}{\partial s} [\hat{T}_i(c_i) - \hat{Q}_i(c_i)(K(s) + s)] \bigg|_{s=c} \\
= \hat{\pi}_i(c_i|c_i) + [-\hat{Q}_i(c_i)(K'(c_i) + 1)] \bigg|_{s=c} \\
= \hat{\pi}_i(c_i|c_i) + [-\hat{Q}_i(c_i)(K'(c_i) + 1)] \\
\]

By Proposition 1, \( \hat{\pi}'(c_i) = -\hat{Q}_i(c_i)(K'(c_i) + 1) \), so now we have
\[
\hat{\pi}'(c_i) = -\hat{Q}_i(c_i)(K'(c_i) + 1) = \hat{\pi}_i(c_i|c_i) + [-\hat{Q}_i(c_i)(K'(c_i) + 1)] \\
\]
or
\[
\hat{\pi}_i(c_i|c_i) = 0 \text{ almost everywhere.} \\
\]
So, we can subtract this from the integrand in (9) to get
\[
\int_{c_i}^{c_i} \left[ \hat{\pi}_i(s|c_i) - \hat{\pi}_i(s|s) \right] ds < 0 \tag{10} \\
\]
The integrand here in (10) is equivalent to the integral \( \int_s^{c} (\hat{\pi}_{i|2}(s|t)) dt \), where \( \hat{\pi}_{i|2} \) denotes the total derivative of \( \hat{\pi}_i \) with respect to the elements of \( t \). Now we have
\[
\int_{c_i}^{c_i} \int_s^{c} (\hat{\pi}_{i|2}(s|t)) dt ds < 0 \tag{11} \\
\]
Now,
\[
\hat{\pi}_{i|2}(s|t) = \frac{\partial}{\partial t} \hat{\pi}_i(s|t) \\
= \frac{\partial}{\partial t} [\hat{T}'_i(s) - \hat{Q}'_i(s)(K'(t_i) + t_i)] \\
= -\hat{Q}'_i(s)(K'(t_i) + 1) \\
\]
Now \( (K'(t_i) + 1) \geq 0 \) because \(-1 \leq K'(t_i) \leq 0 \), and \( \hat{Q}'_i(s) \leq 0 \) almost everywhere by Proposition 2. So \( \hat{\pi}_{i|2}(s|t) \geq 0 \) almost everywhere. This contradicts the inequality (11), which in turn contradicts condition (3.2) that (IC) is violated. So it must be that (IC) holds. \( \square \)
With the above propositions, we have shown that the incentive compatibility constraint (IC) is equivalent to two simple conditions on \( \hat{\pi}_i'(c_i) \) and \( \hat{Q}_i(c_i) \). In addition, the fact that \( \hat{\pi}_i(c_i) \) is nonincreasing in \( c_i \) combined with the (IR) constraint, shows that in cases where \( \hat{\pi}_i(\hat{c}) = 0 \) for \( \hat{c} < c_{\text{max}} \), \( \hat{\pi}_i(c_i) = 0 \) for all \( c_i \in [\hat{c}, c_{\text{max}}] \). Thus, we have established:

**Proposition 4.** If for any feasible solution to (BP), the (IR) constraint is satisfied as an equality for some \( \hat{c} < c_{\text{max}} \), then \( \hat{\pi}_i(c_i) = 0 \) for all \( c_i \in [\hat{c}, c_{\text{max}}] \).

This obviously means that if the agent makes a zero profit for any cost parameter, then he also makes a zero profit for any higher cost parameter. Note that this distinguishes our result from that of Lewis and Sappington, in which the profits are (generally) zero at \( c_{\text{min}} \) and \( c_{\text{max}} \), and in a nontrivial interval around a point \( \hat{c} \) strictly between \( c_{\text{min}} \) and \( c_{\text{max}} \), but positive outside this interval.

Note also that in the McCall model, \( \hat{c} = R \), and the zero-profit region is where production costs are greater than outside revenue, that is, where \( c_i \in [R, c_{\text{max}}] \).

### 4 The Optimal Auction

Having described the set of feasible contracts, we now solve for the optimal auction. First, recall the objective function for the buyer:

\[
E \left[ V \sum_{i=1}^{N} Q_i(\tilde{c}_i | \tilde{c}_{-i}) - \sum_{i=1}^{N} T_i(\tilde{c}_i | \tilde{c}_{-i}) \right]
\]  

(12)

Note that by Myerson’s (1981) Revelation Principle, we may restrict our attention to direct revelation mechanisms in which truth-telling is a Bayesian Nash Equilibrium strategy for all agents. That is, we may need to consider only those mechanisms in which \( \tilde{c}_i = c_i \ \forall \ i = 1, \ldots, N \) in equilibrium.

Now, the definition of profit given in Eq. (3) implies that

\[
T_i(c_i | c_{-i}) = \pi_i(c_i) + Q_i(c_i | c_{-i})(K(c_i) + c_i)
\]
so we can rewrite the objective function (12) as

$$
E \left[ V \sum_{i=1}^{N} Q_i(c_i|c_{-i}) - \sum_{i=1}^{N} Q_i(c_i|c_{-i})(K(c_i) + c_i) - \sum_{i=1}^{N} \pi_i(c_i) \right] \tag{13}
$$

To simplify this function, we calculate $E[\pi_i(c_i)]$ in terms of the decision variables $Q_i(\cdot)$ and $T_i(\cdot)$. Note that the expectation is taken over all $N$ values of $c_i$ ($i = 1, \ldots, N$).

$$
E[\pi_i(c_i)] = \int_{c_{\min}}^{c_{\max}} \cdots \int_{c_{\min}}^{c_{\max}} \left( \int_{c_{\min}}^{c_{\max}} \pi_i(c_i) f(c_i) dc_i \right) f_{-i}(c_{-i}) dc_{-i} \tag{14}
$$

We calculate the innermost integral using integration by parts, where the parts are

$$
u := -\pi_i(c_i) \Rightarrow du = -\pi_i'(c_i) dc_i = Q_i(c_i|c_{-i})(K'(c_i) + 1) dc_i$$

$$dv := -f(c_i) dc_i \Rightarrow v = (1 - F(c_i))$$

Thus,

$$
\int_{c_{\min}}^{c_{\max}} \pi_i(c_i) f(c_i) dc_i = -\pi_i(c_i)(1 - F(c_i))|_{c_{\min}}^{c_{\max}} - \int_{c_{\min}}^{c_{\max}} Q_i(c_i|c_{-i})(K'(c_i) + 1)(1 - F(c_i)) dc_i
$$

$$= -\pi_i(c_{\max}) (1 - F(c_{\max})) + \pi_i(c_{\min}) (1 - F(c_{\min}))$$

$$- \int_{c_{\min}}^{c_{\max}} Q_i(c_i|c_{-i})(K'(c_i) + 1) \frac{1 - F(c_i)}{f(c_i)} f(c_i) dc_i$$

$$= \pi_i(c_{\min}) - E_{c_i} \left[ Q_i(c_i|c_{-i})(K'(c_i) + 1) \frac{1 - F(c_i)}{f(c_i)} \right]$$

Continuing from (14),

$$
E[\pi_i(c_i)] = \int_{c_{\min}}^{c_{\max}} \cdots \int_{c_{\min}}^{c_{\max}} \left( \pi_i(c_{\min}) - E_{c_i} \left[ Q_i(c_i|c_{-i})(K'(c_i) + 1) \frac{1 - F(c_i)}{f(c_i)} \right] \right) f_{-i}(c_{-i}) dc_{-i}
$$

$$= E_{c_{-i}} \left[ \pi_i(c_{\min}) - E_{c_i} \left[ Q_i(c_i|c_{-i})(K'(c_i) + 1) \frac{1 - F(c_i)}{f(c_i)} \right] \right]
$$

$$= \pi_i(c_{\min}) - E_{c_i} \left[ Q_i(c_i|c_{-i})(K'(c_i) + 1) \frac{1 - F(c_i)}{f(c_i)} \right] \tag{15}
$$
Substituting (15) into (13), the objective function becomes
\[
E \left[ V \sum_{i=1}^{N} Q_i(c_i|c_{-i}) - \sum_{i=1}^{N} Q_i(c_i|c_{-i})(K(c_i) + c_i) - \sum_{i=1}^{N} \pi_i(c_{\min}) 
+ \sum_{i=1}^{N} \left( Q_i(c_i|c_{-i})(K'(c_i) + 1) \frac{1 - F(c_i)}{f(c_i)} \right) \right] 
= E \left[ \sum_{i=1}^{N} Q_i(c_i|c_{-i}) \left( V - (K(c_i) + c_i) + (K'(c_i) + 1) \frac{1 - F(c_i)}{f(c_i)} \right) - N\pi_i(c_{\min}) \right] 
\]

Define the preliminary adjusted cost \( j(c_i) \) as follows:
\[
j(c_i) := (K(c_i) + c_i) - (K'(c_i) + 1) \frac{1 - F(c_i)}{f(c_i)} \quad (16)
\]

We can now rewrite (BP) as
\[
\max_{Q_i(.), T_i(.)} E \left[ \sum_{i=1}^{N} Q_i(c_i|c_{-i}) (V - j(c_i)) - N\pi_i(c_{\min}) \right] \quad (BP')
\]
subject to the same constraints as the original problem.

Problem (BP') is a linear program, and thus generally has a corner solution. It seems clear that the objective function is maximized when \( Q_i(c_i|c_{-i}) = 1 \) for the \( c_i \) that minimizes \( j(c_i) \) (assuming all \( c_i \)'s are distinct) and \( Q_i(c_i|c_{-i}) = 0 \) otherwise. However, this is in fact the case only when \( K'(c_i) < 0 \) for that value of \( c_i \).

To see this, first recall from Section 3.1 that \( K'(c_i) < 0 \), except where \( K(c_i) = 0 \), in which case \( K'(c_i) = 0 \). Therefore, if there exists a value of \( \hat{c} \in [c_{\min}, c_{\max}] \) such that \( K'(c_i) = 0 \), then \( K(c_i) = 0 \) for all \( c_i \geq \hat{c} \). Since we are only considering values of \( c_i \) in the closed interval \([c_{\min}, c_{\max}]\), there must exist a value \( \hat{c} \) of \( c_i \) which is the smallest such value. Let \( \hat{c} \) denote this smallest value. For values of \( c_i < \hat{c} \), \( (K(c_i) + c_i) \) is nondecreasing (since \(-1 \leq K'(c_i) \leq 0\)) and \( (K'(c_i) + 1) \) is between 0 and 1 for the same reason, and is nonincreasing (since \( K'' \leq 0 \)). Also, \( \frac{1 - F(c_i)}{f(c_i)} \) is nonincreasing because the distribution satisfies MHRC. The net effect is that \( j(c_i) \) is a nondecreasing function of \( c_i \) for \( c_i < \hat{c} \).

However, for values of \( c_i > \hat{c} \), \( K(c_i) = 0 \) because the outside option is not exercised (recall that the outside profit \( K(c_i) \) is constrained to be nonnegative). Since
\( K(c_i) = 0 \) for all \( c_i > \hat{c} \), clearly \( K'(c_i) = 0 \) for all \( c > \hat{c} \) as well. The function \( j(c_i) \)

is still nondecreasing for \( c_i > \hat{c} \), but \( \lim_{t \uparrow \hat{c}} j(t) > \lim_{t \uparrow \hat{c}} j(t) \). Therefore, \( j(c_i) \)
is discontinuous precisely at the point \( \hat{c} \).

Another way to understand this is to note that \( K''(c_i) \leq 0 \) almost everywhere, which means of course that \( K'(c_i) \) is nonincreasing almost everywhere. For \( c_i < \hat{c} \), \n
\( K'(c_i) \in [-1, 0) \) and nonincreasing. Yet, for \( c_i > \hat{c} \), \( K'(c_i) \) is constrained to its \textit{maximum} value, 0. Therefore, \( K'(c_i) \) is discontinuous at \( \hat{c} \), which implies that \( j(c_i) \)
is discontinuous at \( \hat{c} \) as well.

Note, however, that since \( K(c_i) \) is continuous and \( \lim_{t \uparrow \hat{c}} K'(t) \geq -1 \),

\[
\lim_{t \uparrow \hat{c}} j(t) = \lim_{t \uparrow \hat{c}} \left( (K(c_i) + c_i) - (K'(c_i) + 1) \frac{1-F(c_i)}{f(c_i)} \right)
\]

\[
= (K(\hat{c}) + \hat{c}) - \frac{1-F(\hat{c})}{f(\hat{c})} \left[ \left( \lim_{t \uparrow \hat{c}} K'(c_i) \right) + 1 \right]
\]

\[
\leq (K(\hat{c}) + \hat{c})
\]

\[
= \hat{c}
\]

Therefore, to get around this problem, we can define the \textit{adjusted cost} \( J(c_i) \) as follows:

\[
J(c_i) := \begin{cases} 
  j(c_i) & \text{if } c_i \leq \hat{c} \\
  c_i & \text{if } c_i > \hat{c}
\end{cases} \tag{17}
\]

The adjusted cost \( J(c_i) \) is thus \( j(c_i) \) where \( K'(c_i) < 0 \), and \( K(c_i) + c_i = c_i \) where \( K'(c_i) = 0 \). From the limit computed above, we can see that \( J(c_i) \) is a nondecreasing function of \( c_i \).

Note that \( J(c_i) \) is analogous to the “virtual type” or “priority level” used by Myerson (1981). We can apply a theorem from Myerson to determine that the optimal transfers are given by

\[
\hat{T}_i(c_i) = Q_i(c_i)(K(c_i) + c_i) + \int_{c_i}^{c_{max}} (K'(s) + 1)\hat{Q}_i(s)ds \tag{18}
\]

If \( c_i > \hat{c} \), \( K(c_i) = 0 \) and \( K'(s) = 0 \) for all \( s > c_i \), so this becomes

\[
\hat{T}_i(c_i) = Q_i(c_i)c_i + \int_{c_i}^{c_{max}} \hat{Q}_i(s)ds \tag{19}
\]
This means that the firm chosen is paid an amount equal to its opportunity cost plus an information rent, which implicitly takes into account the probability the firm would have lost the auction if its bid had been higher than its true cost. Note that this information rent is zero for a firm whose cost \( c_i = c_{\text{max}} \).

We have thus shown:

**Proposition 5.** The optimal solution to \((BP')\) is given by

\[
Q_i(c_i|c_{-i}) = \begin{cases} 
1 & \text{if } J(c_i) < \min_{k \neq i}(J(c_k)) \\
\in [0, 1] & \text{if } J(c_i) = \min_{k \neq i}(J(c_k)) \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (20)

and

\[
T_i(c_i|c_{-i}) = Q_i(c_i)(K(c_i) + c_i) + \int_{c_i}^{c_{\text{max}}} (K'(s) + 1)\hat{Q}_i(s)ds
\]  \hspace{1cm} (21)

unless \( J(c_i) > V \) for all \( c_i \), in which case \( Q_i(c_i|c_{-i}) = 0 \) and \( T_i(c_i|c_{-i}) = 0 \) for all \( c_i \).

Of course, in most realistic examples of this auction, firms will actually be paid \( Q_i(c_i|c_{-i})T_i(c_i|c_{-i}) \) rather than \( T_i(c_i|c_{-i}) \); however, as noted in Section 3.1, this is without loss of generality.

### 4.1 Examples

As an example, we compute the optimal solution for the case where \((BP')\) corresponds to the McCall (1970) model. Here we have

\[
K(c_i) = \max(R - c_i, 0)
\]

\[
K'(c_i) = \begin{cases} 
-1 & \text{if } c < R \\
0 & \text{if } c > R
\end{cases}
\]

In this case, \( j(c_i) = (R - c_i) + c_i - (-1 + 1)\frac{1 - F(c_i)}{f(c_i)} = R \). Therefore,

\[
J(c_i) := \begin{cases} 
R & \text{if } c_i \leq R \\
c_i & \text{if } c_i > R
\end{cases}
\]
Note that in this particular case, $J(c_i)$ is continuous at $\hat{c} = R$.

Since $J(c_i) = R$ for all $c_i \leq R$, no differentiation is made among firms with cost less than outside revenue. This is the same type of pooling that occurs in Lewis and Sappington’s countervailing incentives model. In McCall’s model, it presents itself as the expected transfer being equal to $R$ whenever the buyer chooses a firm with $c_i \leq R$. In our model, such a firm receives a transfer of

$$T_i(c_i|c_{-i}) = R + \left[1 - (1 - F(R))^{N-1}\right]$$

and makes a positive expected profit of

$$\hat{\pi}_i(c_i) = R - c_i + \left[1 - (1 - F(R))^{N-1}\right]$$

A firm with higher costs ($c_i > R$), if chosen, is paid its cost plus a smaller premium, that is,

$$T_i(c_i|c_{-i}) = c_i + \left[1 - (1 - F(c_i))^{N-1}\right]$$

and so earns a lower profit than a firm with $c_i < R$, even though it receives a higher transfer. So the buyer’s cost is higher, but the firm earns a lower information rent.

Note that the transfer to a firm with a particular cost parameter in this case is strictly higher than that computed by McCall. This is due to the fact that McCall implicitly assumes an infinite number of potential bidders, and we explicitly include the number of potential bidders in the model. However, the buyer’s expected cost in our case may be lower, because the firm chosen (or equivalently, the values of $Q_i(c_i|c_{-i})$) is not necessarily the same as the one chosen by a buyer who always chooses the lowest bidder for an CPIF contract, as in McCall’s description of a hypothetical government’s behavior. In other words, a government which behaves according to McCall’s description uses his model, but, as McCall proves, does not implement the optimal auction. Rather, it may choose a firm that receives a higher transfer than the optimal firm, even though it receives a lower information rent.
As another example, consider the model of McAfee and McMillan (1986). In this model, it is implicitly assumed that the outside option is zero for all firms. This is equivalent to setting $R = 0$, which implies that $\hat{c} = 0$. Therefore, $J(c_i) = c_i$ for all $c_i > 0$, and the lowest bidder is always chosen. This is exactly the result they find.

5 Conclusion

We have shown that in a simple procurement model, the optimal policy is for the buyer to choose the bidder who minimizes the "adjusted cost" function given by Eq. (17), and pay a transfer based on that bidder’s revealed cost parameter given by Eq. (18). This function trades off the countervailing incentives to overstate low costs in order to share in an incentive fee, and to understate high costs in order to increase one’s probability of winning the contract. This is equivalent to balancing moral hazard and adverse selection, at the cost of paying all but the highest cost bidder a positive information rent. This information rent increases as the seller’s cost and the buyer’s expenditure decrease.

More work is necessary to determine how these results might or might not change in the presence of risk-averse agents or agents whose information about their own costs is still imperfect, but better than the principal’s.
References


